**20.320 Final Write Up**

**SIR-X epidemic model considering psychological influence**

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**Introduction**

The extent to which SIR-X epidemic modelling dynamics predicted the spread of confirmed COVID-19 cases, *C(t)*, in the most affected mainland China provinces was demonstrated in Part 1. The model captured the intermediary superlinear growth phase for all of the most effected mainland China provinces, however, saturating behavior after 9 February was not completely captured for all provinces. However, the model uncovered some underlying factors implicated in the reduction of new confirmed cases during mid-February. As expected, the maximum rate of *C(t)* accumulation occurred when the population was represented by the highest proportion of infected individuals, *I(t)*. After the infectious population reached its maximum, it exhibited exponential decay, and the rate of *C(t)* accumulation was thus driven to zero.

The dynamics responsible for the exponential depletion of *I(t)* are not fully understood. Part 1 implicated the role of containment strategies in the saturating behavior of *C(t)* through their mediated reduction of the susceptible population. However, fitted rate constants, and , may only give a glimpse of a larger picture. The magnitudes of and gave very little insight into the province-specific effects of containment measures. Both values would be conceivably small if the confirmed case count remained much smaller than the total population under the assumption that *X(t)* is proportional to *C(t)*. The aforementioned assumption implies that all quarantined, infectious individuals decide to get tested and all test positive. However, this does not explain asymptomatic individuals in the quarantined population that would presumably have little incentive to get tested. This could be justified a number of ways, for example by assuming that everyone effected by containment gets tested, but that would also require the assumption that only those effected by containment get tested. Regardless of their justifications, the assumptions made in the previously considered SIR-X model reduce the interpretive value of the fitted parameters.

This, in turn, motivated modification of the epidemic model with the hopes of leveraging the ability to uncover previously unconsidered factors that may influence the province-specific response to outbreak.

**Model**

The number of infectious individuals plays a key role in determining the transmission rate of a disease. The behavioral change of susceptible individuals in response to an accumulating infectious population can be represented through the introduction of an infectious force, , where .

(3)

(4)

(2)

(1)

The function is nonmonotone, meaning is positive when is small and negative when is large. To examine the effect of multiple nonlinear transmission rates, is factorized into ; 1 can be interpreted as the “psychological” effect of an accumulating epidemic on people’s willingness to make social contacts (*1*). When , the nonmonotone incidence rate becomes the bilinear incidence rate . Contact inhibition by means of is distinct from containment measures because an individual may still participate in some social contacts, albeit fewer considering their risk assessment, assumed to increase alongside the number of infecteds. and will both be considered among other modified forms, the cited equations required modification as does not represent the number of infecteds in this model (*1*, *2*).

The public response to outbreak is not likely constant. Perceived outbreak severity has been shown to have a large effect on the public response to disease outbreak (*3*). Coefficients representing the psychological effects of risk assessment on containment or contact rates may represent the public’s attitude towards outbreak.

The psychological crowding effect was also examined in the accumulation of the quarantine population by means of .

and will also both be considered. This model assumes that the population’s decision to obey containment policies is dependent on the number of people observed to be participating in social contacts. If an individual does not notice anyone else breaking containment measures, the model assumes that they are less likely to disobey containment measures themselves. Multiple forms of and were tested. and were the only forms that consistently fit *C(t)* data.

**Conclusion**

The ability of SIR-X epidemic dynamics supplemented with behavioral effects to model confirmed cases in some of mainland China’s most effected provinces is demonstrated in Fig. 1, 2. As established in part 1, the exponential depletion of infecteds drove the accumulation of new cases to zero. This saturation behavior was captured in all of the examined provinces with the implementation of and (Fig. 2) and alone (Fig.3).

The goal of this implementation was to identify a relationship between the public’s response to the spread of disease and transmission characteristics. This was not possible by means of interpreting and alone given their dependence on population size and testing breadth. By investigating how these parameters are influenced by behavior, I hoped to look beyond the inherent factors influencing epidemic spread (i.e. population) to gain insight into how agency may affect the outcome of novel disease spread.

The implementation of behavioral effect into the SIR-X model did not ultimately yield substantial information surrounding the province-specific depletion of *X(t)*. However, *I(t)/X(t)* at their maximum values are correlated with . As approaches one, becomes more nonmonotone (Fig. 4). This suggests that provinces with higher , representing the strength of contact moderation in response to perceived outbreak severity, are more negatively affected by outbreak. This result is quite intuitive: if the public is not worried about the spread of infection at early stages and bases their risk assessment on current disease prevalence, an early accumulation of infecteds could increase the maximum outbreak severity before saturation. The major caveat to this correlation is the variability of *I(t)*. More samples should be taken in future works to determine the reproducibility of this tentative finding.

When both and were implemented (Fig. 2), the beforementioned correlation is moves out of statistical significance. Interestingly, values of between provinces were most similar out of any of the other coefficients besides (Table 2,3). Furthermore, was much lower than in both implementations (Table 1,2). Low behavioral effect could indicate that containment measures were followed when assigned compared to being based on public behavior. Furthermore, was fit below 0.08 for all provinces, even when only was considered (Fig. 3, Table 3).

In conclusion, though the model did not definitively identify underlying factors responsible for province-specific responses to the preliminary COVID-19 outbreak, the model suggested the importance of making empirically-based precautionary decisions.

**Chart, line chart

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**Fig 1. The SIR-X compartmental epidemic model predicted confirmed cases with the implementation of infectious force**

Laboratory confirmed cases, *C(t)* were processed as described in part 1. Eqs. 1-4 were also parametrized and solved as described in part 1. < 1 to ensure a nonmonotone infectious force.

The SIR-X model captured the phases of initial exponential growth, the following subexponential growth, and the saturating behavior of *C(t)* in all provinces except for Anhui. The size of *I(t)* relative to *X(t)* at saturation proportional to (R = 0.79, *p = 0.021,* Pearson correlation).

Graphical user interface, chart, line chart

Description automatically generated

**Fig. 2. The implementation of the crowding effect on transmission and containment rates improved the SIR-X epidemic model’s predictive accuracy**

Laboratory confirmed cases, *C(t)* were processed as described in part 1. Eqs. 1-4 were also parametrized and solved as described in part 1. < 1 to ensure a nonmonotone psychological effect was applied to transmission and quarantine rates. when left unbounded was fit to just above 1 for some provinces. This was likely to compensate for the initial low quarantine rates when S was close to unity. The SIR-X model captured the phases of initial exponential growth, the following subexponential growth, and the saturating behavior of *C(t)* in all provinces.

Chart, line chart

Description automatically generated**Fig. 2. The implementation of the crowding effect on containment rates alone**

Laboratory confirmed cases, *C(t)* were processed as described in part 1. Eqs. 1-4 were also parametrized and solved as described in part 1. The SIR-X model captured the phases of initial exponential growth, the following subexponential growth, and the saturating behavior of *C(t)* in all provinces was fit to similar values for all provinces, see Table 3.

Chart, line chart, histogram

Description automatically generated

days since Jan. 20th

**Fig. 4. The monotone character of the infectious force function depends on**

The curves represent over time, equivalent to the infectious force function . The red curve represents in Anhui, with a high of 0.903. The black curve represents in Guangdong with a lower of 0.578. was integrated alongside Eqs. 1-4 then the observed curves were derived.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Providence |  |  |  |  | *I0/X0* |  |
| Guangdong | 0.806 | 0.392 | 0.058 | 0.218 | 0.241 | 0.578 |
| Henan | 0.799 | 0.392 | 0.044 | 0.245 | 1.559 | 0.631 |
| Zhejiang | 0.817 | 0.049 | 0.193 | 2.671e-11 | 0.115 | 0.427 |
| Hunan | 0.776 | 0.216 | 0.098 | 9.613e-07 | 3.696 | 0.715 |
| Anhui | 0.773 | 0.254 | 0.064 | 3.136e-07 | 30.974 | 0.903 |
| Jiangxi | 0.794 | 0.392 | 0.052 | 0.180 | 1.997 | 0.606 |
| Jiangsu | 0.787 | 0.049 | 0.130 | 2.319e-14 | 4.250 | 0.751 |
| Chongqing | 0.785 | 0.266 | 0.081 | 7.530e-06 | 5.696 | 0.897 |

**Table 1 Fitted parameter values without considering the crowding effect on containment rates.**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Providence |  |  |  |  | *I0/X0* |  |  |
| Guangdong | 0.650 | 0.110 | 0.011 | 2.337e-14 | 1.593 | 0.950 | 0.044 |
| Henan | 0.753 | 0.392 | 0.010 | 0.012 | 8.264 | 0.830 | 0.077 |
| Zhejiang | 0.647 | 0.049 | 0.012 | 4.333e-14 | 3.226 | 0.932 | 0.043 |
| Hunan | 0.687 | 0.161 | 0.011 | 1.393e-09 | 8.958 | 0.949 | 0.051 |
| Anhui | 0.682 | 0.281 | 0.009 | 3.945e-12 | 76.939 | 0.991 | 0.064 |
| Jiangxi | 0.722 | 0.173 | 0.007 | 0.041 | 3.890 | 0.845 | 0.060 |
| Jiangsu | 0.632 | 0.178 | 0.009 | 2.277e-13 | 33.428 | 0.999 | 0.050 |
| Chongqing | 0.645 | 0.246 | 0.010 | 0.009 | 8.533 | 0.999 | 0.060 |

**Table 2 Fitted parameter values with nonlinear transmission and containment rates**

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Providence |  |  |  |  | *I0/X0* |  |
| Guangdong | 0.684 | 0.110 | 0.012 | 2.337e-14 | 1.593 | 0.050 |
| Henan | 0.848 | 0.338 | 0.010 | 0.009 | 8.568 | 0.068 |
| Zhejiang | 0.694 | 0.049 | 0.011 | 2.360e-14 | 3.226 | 0.038 |
| Hunan | 0.550 | 0.064 | 0.013 | 4.276e-14 | 12.059 | 0.053 |
| Anhui | 0.752 | 0.345 | 0.010 | 6.838e-08 | 83.611 | 0.074 |
| Jiangxi | 0.810 | 0.117 | 0.006 | 0.037 | 3.644 | 0.050 |
| Jiangsu | 0.550 | 0.096 | 0.014 | 2.256e-14 | 29.657 | 0.067 |
| Chongqing | 0.550 | 0.181 | 0.010 | 3.083e-14 | 12.632 | 0.050 |

**Table 3 Fitted parameter values with nonlinear containment rates**

**Bibliography**

1. D. Xiao, S. Ruan, Global analysis of an epidemic model with nonmonotone incidence rate. *Mathematical Biosciences*. **208**, 419–429 (2007).

2. J. Cui, Y. Sun, H. Zhu, The Impact of Media on the Control of Infectious Diseases. *J Dyn Diff Equat*. **20**, 31–53 (2008).

3. Some epidemiological models with nonlinear incidence | SpringerLink, (available at https://link.springer.com/article/10.1007/BF00160539).